

INTERSTELLAR RADIO LINKS ENHANCED BY EXPLOITING THE SUN AS A GRAVITATIONAL LENS

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ABSTRACT. The gravitational lens of the Sun is an astrophysical phenomenon predicted by Einstein's general theory of relativity. It implies that, if we can send a probe along any radial direction away from the Sun up to the minimal distance of 550 AU and beyond, the Sun's mass will act as a huge magnifying lens, letting us "see" detailed radio maps of whatever may lie on the other side of the Sun even at very large distances. The recent book by this author (Claudio Maccone, *Deep Space Flight and Communications*, 414 pages, Praxis-Springer, 2009) studies such future FOCAL space missions to 550 AU and beyond.

In this paper, however, we want to study another possibility yet: how to INCREASE the future interstellar radio links between the solar system and any future interstellar probe by utilizing the gravitational lens of the Sun as a huge antenna.

In particular, we study the Bit Error Rate (BER) across interstellar distances with and without using the gravitational lens effect of the Sun.

The conclusion is that only when we will exploit the Sun as a gravitational lens we will be able to communicate with our own probes (or with nearby Aliens) across the distances of even the nearest stars to us in the Galaxy, and that at a reasonable Bit Error Rate.

Keywords: Sun gravitational lens, space probe, special relativity, propulsion, telecommunications.

1. INTRODUCTION

The gravitational focusing effect of the Sun is one of the most amazing discoveries produced by the general theory of relativity. The first paper in this field was published by Albert Einstein in 1936 (ref. [1]), but his work was virtually forgotten until 1964, when Sydney Liebes of Stanford University (ref. [2]) gave the mathematical theory of gravitational focussing by a galaxy located between the Earth and a very distant cosmological object, such as a quasar.

In 1978 the first "twin quasar" image, caused by the gravitational field of an intermediate galaxy, was spotted by the British astronomer Dennis Walsh and his colleagues. Subsequent discoveries of several more examples of gravitational lenses eliminated all doubts about gravitational focussing predicted by general relativity.

Von Eshleman of Stanford University then went on to apply the theory to the case of the Sun in 1979 (ref. [3]). His paper for the first time suggested the possibility of sending a spacecraft to 550 AU from the

Sun to exploit the enormous magnifications provided by the gravitational lens of the Sun, particularly at microwave frequencies, such as the hydrogen line at 1420 MHz (21 cm wavelength). This is the frequency that all SETI radioastronomers regard as "magic" for interstellar communications, and thus the tremendous potential of the gravitational lens of the Sun for getting in touch with alien civilizations became obvious.

The first experimental SETI radioastronomer in history, Frank Drake (*Project Ozma*, 1960), presented a paper on the advantages of using the gravitational lens of the Sun for SETI at the *Second International Bioastronomy Conference* held in Hungary in 1987 (ref. [4]), as did Nathan "Chip" Cohen of Boston University (ref. [5]). Non-technical descriptions of the topic were also given by them in their popular books (refs. [6] and [7]).

However, the possibility of planning and funding a space mission to 550 AU to exploit the gravitational lens of the Sun immediately proved a difficult task. Space scientists and engineers first turned their attention to this goal at the June 18, 1992, *Conference on Space Missions and Astrodynamics* organized in

Turin, Italy, lead by me (see Figure 1.1). The relevant Proceedings were published in 1994 in the *Journal of the British Interplanetary Society* (ref. [8]). Meanwhile, on May 20, 1993 I also submitted a formal Proposal to the European Space Agency (ESA) to fund the space mission design (ref. [9]). The optimal direction of space to launch the FOCAL spacecraft was also discussed by Jean Heidmann of Paris Meudon Observatory and myself (ref. [10]), but it seemed clear that a demanding space mission like this one should not be devoted entirely to SETI. Things like the computation of the parallaxes of many distant stars in the Galaxy, the detection of gravitational waves by virtue of the very long baseline between the spacecraft and the Earth, plus a host of other experiments would complement the SETI utilization of this space mission to 550 AU and beyond.

The mission was dubbed "SETISAIL" in earlier papers (ref. [11]), and "FOCAL" in the proposal submitted to ESA in 1993.

In the third edition of his book "The Sun as a Gravitational Lens: Proposed Space Missions" (ref. 12), the author summarized all knowledge available as of 2002 about the FOCAL space mission to 550 AU and beyond to 1000 AU. On October 3rd, 1999, this book had already been awarded the Engineering Science Book Award by the International Academy of Astronautics (IAA).

Finally, in March 2009, the new, 400-pages and comprehensive book by the author, entitled "Deep Space Flight and Communications – Exploiting the Sun as a Gravitational Lens" (ref. [19]), was published. This book embodies all the previous material published about the FOCAL space mission and updates it.

2. WHY 550 AU IS THE MINIMAL DISTANCE THAT "FOCAL" MUST REACH

The geometry of the Sun gravitational lens is easily described: incoming electromagnetic waves (arriving, for instance, from the center of the Galaxy) pass *outside* the Sun and pass within a certain distance r of its center. Then the basic result following from the Schwarzschild solution shows that the corresponding *deflection angle* $\alpha(r)$ at the distance r from the Sun center is given by

$$\alpha(r) = \frac{4GM_{Sun}}{c^2 r}. \quad (1)$$

Figure 1 shows the basic geometry of the Sun gravitational lens with the various parameters in the game.

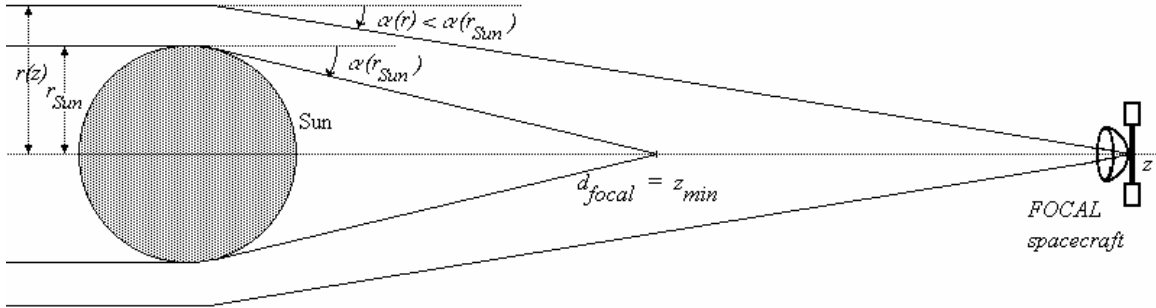


Figure 1. Geometry of the Sun gravitational lens with the minimal focal length of 550 AU (= 3.17 light days = 13.75 times beyond Pluto's orbit) and the FOCAL spacecraft position beyond the minimal focal length.

The light rays, i.e. electromagnetic waves, cannot pass through the Sun's interior (whereas gravitational waves and neutrinos can), so the largest deflection angle α occurs for those rays just grazing the Sun surface, i.e. for $r = r_{Sun}$. This yields the inequality

$$\alpha(r_{Sun}) > \alpha(r) \quad (2)$$

with

$$\alpha(r_{Sun}) = \frac{4GM_{Sun}}{c^2 r_{Sun}}. \quad (3)$$

From the illustration it should be clear that the minimal focal distance d_{focal} is related to the tangent of the maximum deflection angle by the formula

$$\tan(\alpha(r_{Sun})) = \frac{r_{Sun}}{d_{focal}}. \quad (4)$$

Moreover, since the angle $\alpha(r_{Sun})$ is very small (its actual value is about 1.75 arc seconds), the above expression may be rewritten by replacing the tangent by the small angle itself:

$$\alpha(r_{Sun}) \approx \frac{r_{Sun}}{d_{focal}}. \quad (5)$$

Eliminating the angle $\alpha(r_{Sun})$ between equations (3) and (5), and then solving for the minimal focal distance d_{focal} , one gets

$$d_{focal} \approx \frac{r_{Sun}}{\alpha(r_{Sun})} = \frac{r_{Sun}}{\frac{r_{Sun}}{4GM_{Sun}}} = \frac{c^2 r_{Sun}^2}{4GM_{Sun}}. \quad (6)$$

This basic result may also be rewritten in terms the *Schwarzschild radius*

$$r_{Schwarzschild} = \frac{2GM_{Sun}}{c^2}, \quad (7)$$

yielding

$$d_{focal} \approx \frac{r_{Sun}}{\alpha(r_{Sun})} = \frac{r_{Sun}}{\frac{r_{Sun}}{4GM_{Sun}}} = \frac{r_{Sun}^2}{2r_{Schwarzschild}}. \quad (8)$$

Numerically, one finds

$$d_{focal} \cong 542 \text{ AU} \approx 550 \text{ AU} \approx 3.171 \text{ light days}. \quad (9)$$

This is the fundamental formula yielding the minimal focal distance of the gravitational lens of the Sun, i.e. the minimal distance from the Sun's center that the FOCAL spacecraft must reach in order to get magnified radio pictures of whatever lies on the other side of the Sun with respect to the spacecraft position.

Furthermore, a simple, but very important consequence of the above discussion is that ***all points on the straight line beyond this minimal focal distance are foci too***, because the light rays passing by the Sun further than the minimum distance have smaller deflection angles and thus come together at an even greater distance from the Sun.

And the very important astronomical consequence of this fact for the FOCAL mission is that ***it is not necessary to stop the spacecraft at 550 AU. It can go on to almost any distance beyond and focus as well or better***. In fact, the further it goes beyond 550 AU the less distorted the collected radio waves by the Sun Corona fluctuations. The important problem of Corona fluctuations and related distortions is currently being studied by Von Eshleman and colleagues at Stanford University (please refer to Chapter 6 of ref. [19]).

We would like to add here one more result that is very important because it holds well not just for the Sun, but for all stars in general. This we will do without demonstration; that can be found on page 55 of ref. [12]. Consider a spherical star with radius r_{star} and mass M_{star} , that will be called the "focusing star". Suppose also that a light source (i.e. another star

or an advanced extraterrestrial civilization) is located at the distance D_{source} from it. Then ask: how far is the minimal focal distance d_{focal} on the opposite side of the source with respect to the focussing star center? The answer is given by the formula

$$d_{focal} = \frac{r_{star}^2}{\frac{4GM_{star}}{c^2} - \frac{r_{star}^2}{D_{source}}}. \quad (10)$$

This is the key to gravitational focussing for a pair of stars, and may well be the key to SETI in finding extraterrestrial civilizations. It could also be considered for the magnification of a certain source by any star that is perfectly aligned with that source and the Earth: the latter would then be in the same situation as the FOCAL spacecraft except, of course, it is located much further out than 550 AU with respect to the focussing, intermediate star. Finally, notice that equation (10) reduces to equation (6) in the limit $D_{source} \rightarrow \infty$, i.e. (6) is the special case of (10) for light rays approaching the focussing star from an infinite distance.

3. THE HUGE (ANTENNA) GAIN OF THE GRAVITATIONAL LENS OF THE SUN

Having thus determined the minimal distance of 550 AU that the FOCAL spacecraft must reach, one now wonders what's the good of going so far out of the solar system, i.e. how much focussing of light rays is caused by the gravitational field of the Sun. The answer to such a question is provided by the technical notion of "antenna gain", that stems out of antenna theory.

A standard formula in antenna theory relates the antenna gain, $G_{antenna}$, to the antenna effective area, $A_{effective}$, and to the wavelength λ or the frequency ν by virtue of the equation (refer, for instance, to ref. [13], in particular page 6-117, equation (6-241)):

$$G_{antenna} = \frac{4\pi A_{effective}}{\lambda^2}. \quad (11)$$

Now, assume the antenna is circular with radius $r_{antenna}$, and assume also a 50% efficiency. Then, the antenna effective area is obviously given by

$$A_{effective} = \frac{A_{physical}}{2} = \frac{\pi r_{antenna}^2}{2}. \quad (12)$$

Substituting this back into (11) yields the antenna gain as a function of the antenna radius and of the observed frequency :

$$G_{antenna} = \frac{4\pi A_{effective}}{\lambda^2} = \frac{2\pi A_{physical}}{\lambda^2} =$$

$$= \frac{2\pi^2 r_{antenna}^2}{\lambda^2} = \frac{2\pi^2 r_{antenna}^2}{c^2} \cdot \nu^2 . \quad (13)$$

The important point here is that *the antenna gain increases with the square of the frequency*, thus favoring observations on frequencies as high as possible.

Is anything similar happening for the Sun's gravitational lens also? *Yes* is the answer, and the "gain" (one maintains this terminology for convenience) of the gravitational lens of the Sun can be proved to be

$$G_{Sun} = 4\pi^2 \frac{r_{Schwarzschild}}{\lambda} \quad (14)$$

or, invoking the expression (1.2-7) of the Schwarzschild radius

$$G_{Sun} = \frac{8\pi^2 G M_{Sun}}{c^2} \cdot \frac{1}{\lambda} = \frac{8\pi^2 G M_{Sun}}{c^3} \cdot \nu . \quad (15)$$

The mathematical proof of equation (1.3-4) is difficult to achieve. The author, unsatisfied with the treatment of this key topic given in refs. [1], [3] and [13], turned to three engineers of the engineering school in his home town, Renato Orta, Patrizia Savi and Riccardo Tascone. To his surprise, in a few weeks they provided a full proof of not just the Sun gain formula (1.3-4), but also of the focal distance for rays originated from a source at finite distance, equation (1.2-10). Their proof is fully described in ref. [12], and is based on the aperture method used to study the propagation of electromagnetic waves, rather than on ray optics.

Using the words of these three authors' own Abstract, they have "computed the radiation pattern of the [spacecraft] Antenna+Sun system, which has an extremely high directivity. It has been observed that

the focal region of the lens for an incoming plane wave is a half line parallel to the propagation direction starting at a point [550 AU] whose position is related to the blocking effect of the Sun disk (Figure 1). Moreover, a characteristic of this thin lens is that its gain, defined as the magnification factor of the antenna gain, is constant along this half line. In particular, for a wavelength of 21 cm, this lens gain reaches the value of 57.5 dB. Also a measure of the transversal extent of the focal region has been obtained. The performance of this radiation system has been determined by adopting a thin lens model which introduces a phase factor depending on the logarithm of the impact parameter of the incident rays. Then the antenna is considered to be in transmission mode and the radiated field is computed by asymptotic evaluation of the radiation integral in the Fresnel approximation".

One is now able to compute the Total Gain of the Antenna+Sun system, that is simply obtained by multiplying equations the two equations yielding the spacecraft gain proportional to ν^3 and the Sun gain proportional to ν :

$$G_{Total} = G_{Sun} \cdot G_{antenna} = \frac{16\pi^4 G M_{Sun} r_{antenna}^2}{c^5} \cdot \nu^3 \quad (16)$$

Since the total gain increases with the *cube* of the observed frequency, it favors electromagnetic radiation in the microwave region of the spectrum. The table in Figure 2 shows the numerical data provided by the last equation for five selected frequencies: the hydrogen line at 1420 MHz and the four frequencies that the Quasat radio astronomy satellite planned to observe, had it been built jointly by ESA and NASA as planned before 1988, but Quasat was abandoned by 1990 due to lack of funding. The definition of dB is of course:

$$N \text{ dB} = 10 \text{Log}_{10} N = 10 \ln N / \ln 10 .$$

Line	Neutral Hydrogen		<i>OH</i> radical		<i>H₂O</i>
Frequency ν	1420 MHz	327 MHz	1.6 GHz	5 GHz	22 GHz
Wavelength λ	21 cm	92 cm	18 cm	6 cm	1.35 cm
S/C Antenna Beamwidth	1.231 deg	5.348 deg	1.092 deg	0.350 deg	0.080 deg
Sun Gain	57.4 dB	51.0 dB	57.9 dB	62.9 dB	69.3 dB
12-meter Antenna S/C Gain	42.0 dB	29.3 dB	43.1 dB	53.0 dB	65.8 dB
Combined Sun + S/C Gain	99.5 dB	80.3 dB	101.0 dB	115.9 dB	135.1 dB

Table 1: Table showing the gain of the Sun's lens alone, the gain of a 12-meter spacecraft (S/C) antenna and the combined gain of the Sun+S/C Antenna system the at five selected frequencies important in radioastronomy.

4. THE 2009 NEW BOOK BY THE AUTHOR ABOUT THE “FOCAL” SPACE MISSION

In March 2009, the new, 400-pages and comprehensive book by the author, entitled “Deep Space Flight and Communications – Exploiting the Sun as a

Gravitational Lens” (ref. 19), was published. This book embodies all the previous material published about the FOCAL space mission and updated it in view of submitting a formal Proposal to NASA about FOCAL. The front and back covers of this book are reproduced in Figure 3 below.

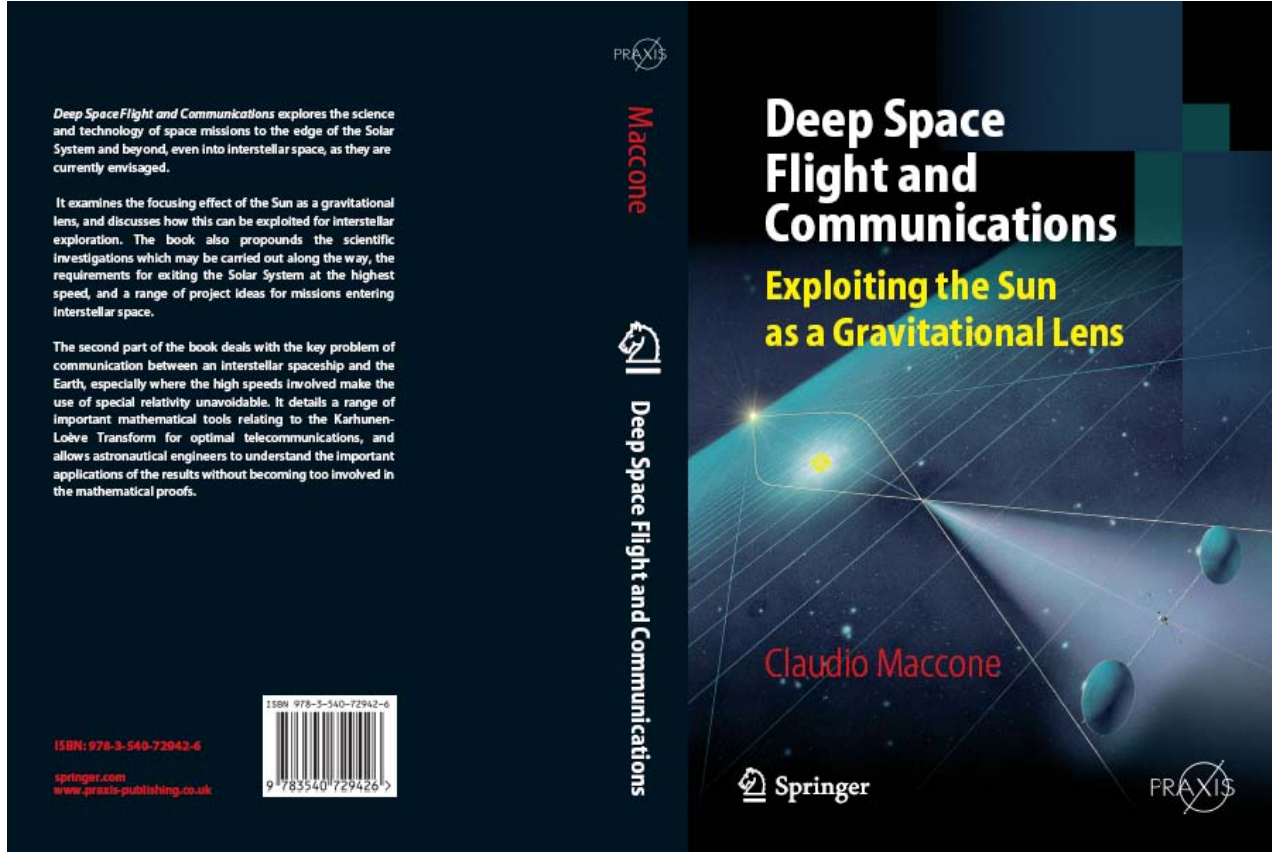


Figure 3. Front and back covers of the author’s new book entitled “Deep Space Flight and Communications – Exploiting the Sun as a Gravitational Lens” published by Springer-Praxis in March 2009, see ref. [19].

5. THE RADIO LINK

The goal of this paper is to prove that only by exploiting the Sun as a gravitational lens we will be able to have reliable telecommunication links across large interstellar distances. In other words, a direct link between different star systems, even if held by virtue of the largest radiotelescopes on Earth, will not be feasible across distances of the order or thousand of light years or more. We want to show that only a FOCAL mission in the direction from Earth opposite to that target star system will ensure a reliable telecommunication link across thousands of light years. Namely, we prove that Bit Error Rate (or BER, see site http://www.en.wikipedia.org/wiki/Bit_error_rate) will be unacceptable already at the distance of Alpha Centauri unless we resort to a supporting FOCAL space mission in the opposite direction from the Sun.

In order to face these problems mathematically, we must first understand the radio link among any two stars, and we think that no neater treatment of this subject exists than the book by “Radio Astronomy” by the late professor John D. Kraus of Ohio State University (ref. [20]), that we follow hereafter.

Consider a radio transmitter that radiates a Power P_t isotropically and uniformly over a bandwidth B_t . Then, at a distance r it produces a flux density given by

$$\frac{P_t}{B_t 4\pi r^2} \quad (17)$$

A receiving antenna of effective aperture A_{er} at a distance r can collect a power given by (17) multiplied by both the effective aperture of the receiving antenna

and its bandwidth, namely the received power P_r is given by

$$P_r = \frac{P_t}{B_t} \frac{A_{er}}{4\pi r^2} B_r \dots \quad (18)$$

It is assumed that the receiving bandwidth B_r is smaller or, at best (in the “matched bandwidths” case) equal to the transmitting bandwidth B_t , that is $B_r \leq B_t$.

So far, we have been talking about an isotropic radiator. But let us now assume that the transmitting antenna has a directivity D , that is an antenna gain in the sense of (11):

$$D = \frac{4\pi A_{et}}{\lambda^2} \quad (19)$$

The received power P_r is then increased by just such a factor due to the directivity of the transmitting antenna, and so (18) must now be replaced by a new equation where the right-hand side is multiplied by such a increased factor, that is

$$P_r = \frac{4\pi A_{et}}{\lambda^2} \cdot \frac{P_t}{B_t} \frac{A_{er}}{4\pi r^2} B_r \dots \quad (20)$$

Rearranging a little, this becomes

$$P_r = \frac{P_t A_{et} A_{er}}{r^2 \lambda^2} \frac{B_r}{B_t} \quad (21)$$

This is the **received signal power** expression. For the matched bandwidths case, i.e. for $B_r = B_t$, this is called the Friis transmission formula, since it was first published back in 1946 by the American radio engineer Harald T. Friis (1893-1976) of the Bell Labs. In space missions, we of course exactly both B_t and B_r and so we construct the spacecraft in such a way the two **bands match exactly**, i.e. $B_r = B_t$. So, for the case of telecommunications with a spacecraft (but not necessarily for the SETI case, we may well assume the matched bandwidths and have (21) reducing to

$$P_r = \frac{P_t A_{et} A_{er}}{r^2 \lambda^2} \quad (22)$$

Let us now rewrite (22) in such a way that we may take into account the gains (i.e. directionalities) of both the transmitting and receiving antennae, that is, in agreement with (11)

$$\left\{ \begin{array}{l} G_t = \frac{4\pi A_{et}}{\lambda^2} \\ G_r = \frac{4\pi A_{er}}{\lambda^2} \end{array} \right. \quad \text{that is} \quad \left\{ \begin{array}{l} A_{et} = \frac{G_t \lambda^2}{4\pi} \\ A_{er} = \frac{G_r \lambda^2}{4\pi} \end{array} \right. \quad (23)$$

Replacing the last two expressions into (22), we find that (22) is turned into

$$P_r = \frac{P_t G_t G_r}{(4\pi)^2 r^2} \lambda^2 \quad .$$

This may finally be rewritten in the more traditional form

$$P_r = \frac{P_t G_t G_r}{L(r, \lambda)} \quad (24)$$

if one defines

$$L(r, \lambda) = (4\pi)^2 \cdot \frac{r^2}{\lambda^2} \quad (25)$$

is the Path Loss (or path attenuation), i.e. the reduction in power density (attenuation) of the electromagnetic waves as they propagates through space. Path loss is a major component in the analysis and design of the link budget of a telecommunication system, see the site http://www.en.wikipedia.org/wiki/Path_loss .

6. BIT ERROR RATE FOR AN “ORDINARY” DIRECT LINK WITH A PROBE AT THE ALPHA CENTAURI DISTANCE

In this section we first define the Bit Error Rate (BER). Then, by virtue of a numerical example, we show that, even at the distance of the nearest star (Alpha Cen. at 4.37 AU) the telecommunications would be impossible by the ordinary powers available today for interplanetary space flight. But in the next section we shall show that the telecommunications would become feasible if we could take advantage of the magnification provided by the Sun’s gravity lens, i.e. if we would send out to 550 AU a FOCAL relay spacecraft for each target star system that we wish to communicate with. And this is the key new result presented in this paper.

So, let us start by defining the Bit Error Rate or BER. In telecommunication theory an **error ratio** is the ratio of the number of bits, elements, characters, or blocks incorrectly received to the total number of bits, elements, characters, or blocks sent during a specified time interval. Among these error ratios, the most commonly encountered ratio is the **bit error ratio** (BER) - also called **bit error rate** - that is the number of erroneous bits received divided by the total number of bits transmitted. At the bit error rate Wikipedia site: http://www.en.wikipedia.org/wiki/Bit_error_rate it is shown that the likelihood of a bit misinterpretation

$$p_e = p(0 | 1)p_1 + p(1 | 0)p_0 \quad (26)$$

(believing that we have received a 0 while it was a 1 or the other way round) is basically given by the “complementary error function” or $\text{erfc}(x)$ as follows

$$BER(d, \nu, P_t) = \frac{1}{2} \operatorname{erfc} \left(\sqrt{\frac{E_b(d, \nu, P_t)}{N_0}} \right) \quad (27)$$

In this equation one has:

- 1) d = distance between the transmitting station on Earth and the receiving antenna in space. For instance, this could be the antenna of a precursor interstellar space probe that was sent out to some light years away.
- 2) ν = frequency of the electromagnetic waves used in the telecommunication link. The higher this frequency, the better it is, since the photons are then more energetic ($E = h \nu$). In today's practice, however, the highest ν for spacecraft links (like the link of the Cassini probe, now at Saturn) are the ones in the Ka band, that is: $\nu_{Ka} \approx 32$ GHz .
- 3) P_t is the power in watts transmitted by the Earth antenna, typically a NASA Deep Space Network antenna 70 meters in diameter.
- 4) The complementary error function $\operatorname{erfc}(x)$ is defined by the integral

$$\operatorname{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^{\infty} e^{-t^2} dt \quad (28)$$

(for more maths, see the relevant Wikipedia site: http://www.en.wikipedia.org/wiki/Complementary_error_function).

- 5) $E_b(d, \nu, P_t)$ is the received energy per bit, that is the ratio

$$E_b(d, \nu, P_t) = \frac{P_r(d, \nu, P_t)}{\text{Bit_rate}} \quad (29)$$

- 6) Finally, N_0 is given by the Boltzmann's constant k multiplied by the noise temperature of space far away from the Sun and from any other star. This "empty space noise temperature" might be assumed to equal, say, to 100 K.

This is the analytical structure of the MathCad code that this author wrote to yield the BER. Let us now consider the input values that he used in practice:

- 1) Suppose that a human space probe has reached the Alpha Cen system at 4.37 light year distance from the Sun: then, $d=4.37$ light years.
- 2) Suppose also that the transmitting antenna from the Earth is a typical NASA Deep Space Network (DSN) antenna having a diameter of 70 meter (like those at Goldstone, Madrid and Canberra), and assume that its efficiency is about 50%.
- 3) Suppose that the receiving antenna aboard the spacecraft is 12 meter in diameter (it might be an inflatable space antenna, as we supposed in ref. [19] for the FOCAL spacecraft) and assume a 50% efficiency.
- 4) Suppose that the link frequency is the Ka band (i.e. 32 GHz), as for the Cassini highest frequency.
- 5) Suppose that the bit rate is 32 kbps = 32000 bit / second. This is the bit rate of ESA's Rosetta

interplanetary spacecraft now on its way to a comet.

- 6) And finally (*this is the most important input assumption*) suppose that the transmitting power P_t is moderate: just 40 watts.

Then:

- 1) The gain of the transmitting NASA DSN antenna (at this Ka frequency) is about 84 dB.
- 2) The gain of the spacecraft antenna is about 69 dB.
- 3) The path loss at the distance of Alpha Cen is 395 dB (a very high indeed path loss with respect to today's interplanetary missions, of course).
- 4) The power received by the spacecraft at that distance is 2.90×10^{-23} watt.
- 5) The received energy per bit (lowered by the noise temperature of the space in between the Sun and Alpha Cen) is 1.3×10^{-37} joule.
- 6) *And finally the BER is 0.49, i.e. there is a 50% probability of ERRORS in the telecommunications between the Earth and the probe at Alpha Cen. if we use such a small transmitting power !!!*

In other words, if these are the telecommunication links between the Earth and our probe at Alpha Cen, then this precursor interstellar mission is *worthless*.

The key point in this example is that, for all calculations, (24) and (25) were used **WITHOUT TAKING THE GAIN OF THE SUN GRAVITY LENS INTO ACCOUNT**, because this was a **DIRECT link and NOT a FOCAL mission**.

7. BIT ERROR RATE AT THE ALPHA CENTAURI DISTANCE ENHANCED BY THE MAGNIFICATION PROVIDED BY THE SUN'S GRAVITY LENS (FOCAL)

The disappointing BER results of the previous section are totally reversed, however, if we suppose that a FOCAL space mission has been previously sent out to 550 AU in the direction opposite to Alpha Cen so that we now have the MAGNIFICATION of the Sun's Gravity Lens playing in the game.

Mathematically, this means that we must introduce a third multiplicative gain at the numerator of (24): the Sun's Gravity Lens GAIN, given by (14) where the Schwarzschild radius of the Sun is given by (7).

This new gain is huge at the Ka band frequency:

$$G_{\text{Sun}}(\nu_{Ka}) = 12444837 \sim 70 \text{ dBs} \quad (30)$$

and so the received power (24) at Alpha Cen, with the usual Earth-transmitted power of just 40 watts becomes

$$P_r = 2.9 \times 10^{-23} \text{ watts} \quad (31)$$

and the relevant BER becomes absolutely acceptable:

$$\boxed{\text{BER} = 0.000000526387845} \quad (32)$$

This should convince anybody that the FOCAL space mission is indispensable to keep the link at interstellar distances equal or higher than Alpha Cen.

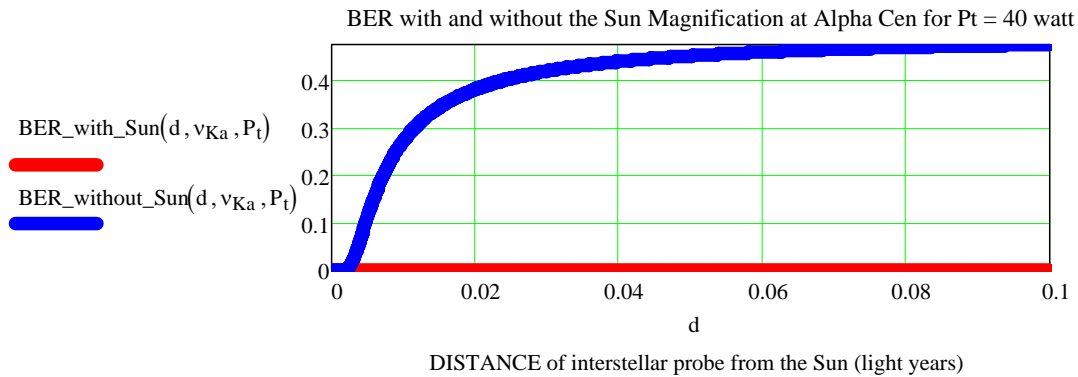


Figure 4. The Bit Error Rate (BER) (upper, blue curve) tends immediately to the 50% value (BER = 0.5) even at moderate distances from the Sun (0 to 0.1 light years) for a 40 watt transmission from a DSN antenna that is a DIRECT transmission, i.e. without using the Sun’s Magnifying Lens. On the contrary (lower red curve) the BER keeps staying at zero value (perfect communications!) if the FOCAL space mission is made, so as the Sun’s magnifying action is made to work.

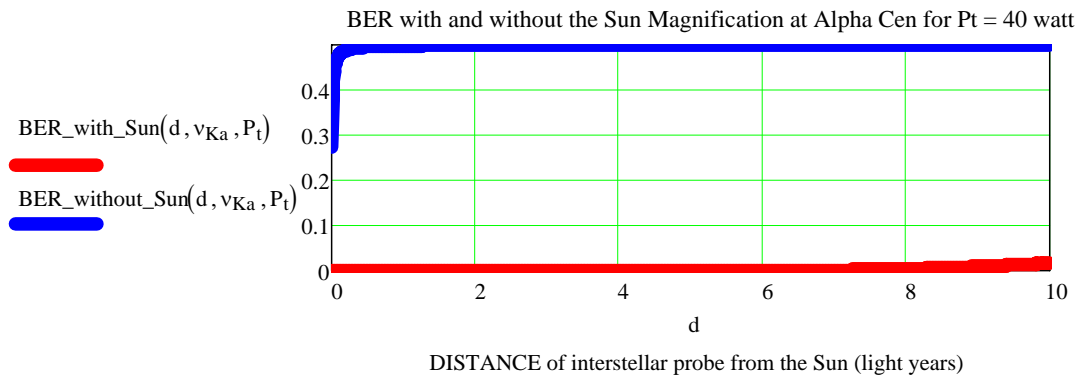


Figure 5. Same as in Figure 4, but for probe distances up to 10 light years. We see that at about 9 light years away the BER curve starts being no exactly flat any more, and starts increasing slowly.

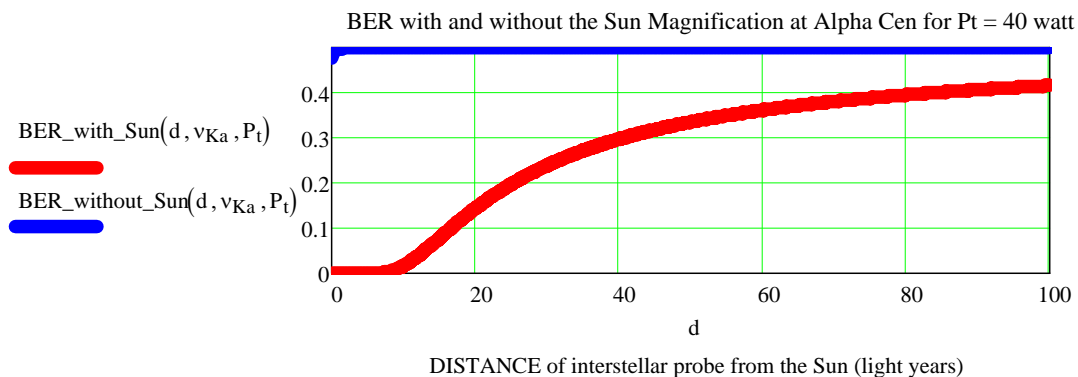


Figure 6. Same as in Figure 5, but for probe distances up to 100 light years. We see that, from 9 light years onward, the Sun-BER increases, reaching the dangerous level of 40% (Sun-BER = 0.4) at about 100 light years. Namely, at 100 light years even the Sun’s Lens cannot cope with this very low transmitted power of 40 watt.

8. THE FANTASTIC “RADIO BRIDGE” BETWEEN THE SUN AND ALPHA CEN A BY USING THE TWO GRAVITATIONAL LENSES OF BOTH JUST MATCHED TO EACH OTHER

In this section we provide one more new result: we define the radio bridge between the Sun and Alpha Cen A by using BOTH gravitational lenses!

In other words, suppose that in future we will be able to send a probe to Alpha Cen A and suppose that we succeed in placing this probe just on the other side of Alpha Cen A with respect to the Sun and at the minimal focal distance typical of Alpha Cen A. This distance is NOT 550 AU obviously because both the radius and the mass of Alpha Cen A are different (actually slightly higher) than the values of the Sun:

$$\begin{cases} r_{Alpha_Cen_A} = 1.227 r_{Sun} \\ M_{Alpha_Cen_A} = 1.100 M_{Sun} \end{cases} \quad (33)$$

Replacing these values into (6) (obviously rewritten for Alpha Cen A), the relevant minimal focal distance is found

$$d_{focal_Alpha_Cen_A} \approx \frac{c^2 r_{Alpha_Cen_A}^2}{4GM_{Alpha_Cen_A}} \approx 749 \text{ AU} . \quad (34)$$

The Schwarzschild radius for Alpha Cen A is given by

$$r_{Schwarzschild_Alpha_Cen_A} = \frac{2GM_{Alpha_Cen_A}}{c^2} = 3.248 \text{ km} \quad \dots(35)$$

And so the gain, provided by (14), turns out to equal

$$\begin{aligned} G_{Alpha_Cen_A}(v_{Ka}) &= 4\pi^2 \frac{r_{Schwarzschild_Alpha_Cen_A}}{\lambda_{Ka}} = \\ &= 13689321 . \end{aligned} \quad (36)$$

That is

$$\boxed{G_{Alpha_Cen_A}(v_{Ka}) \approx 71 \text{ dB}} \quad (37)$$

Incidentally, we chose Alpha Cen A, and not B or C, because it has the highest mass, and so the highest gain, in the whole Alpha Cen triple system. The future telecommunications between the Sun and the Alpha Cen system are thus optimized by selecting Alpha Cen A as the star on the other side of which to place a FOCAL spacecraft at the minimal distance of 750 AU. That FOCAL spacecraft would then easily relay its data anywhere within the Alpha Cen system.

Having found the Alpha Cen A gain (37) we are now able to write the new equation corresponding to (24) for the Sun-Alpha Cen bridge. In fact, we must now put at the numerator of 24 three gains:

1) The Sun gain at 32 GHz,

- 2) The Alpha Cen A gain at 32 GHz, and
- 3) The 12-meter FOCAL antenna gain at 32 GHz raised to the square because there are two such 12-meter antennas: one at 550 AU from the Sun and one at 749 AU from Alpha Cen A, and they must be perfectly aligned with the axis passing thru both the Sun and Alpha Cen A.

Thus, the received power given by (24) now reads

$$P_r = \frac{P_t G_{Sun} G_{Alpha_Cen_A} (G_{12_meter_antenna_at_Ka})^2}{L(r, \lambda)} . \quad \dots(38)$$

where obviously r equals 4.37 light years and λ corresponds to a 32 GHz frequency.

Let us now go back to the BER and replace (38) instead of (24) in the long chain of calculations described in Section 6. Since the received power P_r has now changed, clearly both (29) and (27) yield different numerical results. But now:

- 1) The link frequency has been fixed at 32 GHz (Ka band), and so no longer is an independent variable in the game.
- 2) Also the distance d has been fixed (it is the distance of Alpha Cen A) and so it no longer an independent variable in the game.
- 3) It follows that, in (29) and (27) the only variable to be free to vary is now the transmitted power, P_t .

Let us re-phrase the last sentence in different terms. Practically, we are now studying the BER as a function of the transmitted power P_t only and, physically, this mean that:

- a) We start by inputting very low transmission powers in watts, and find out that the BER is and awful 50%, i.e. the telecommunication between the Sun and Alpha Cen are totally disrupted. This is of course because the energy per bit is so much lower than the empty space noise temperature.
- b) We then increase the transmitted power, at a certain point the BER starts getting smaller than 50%. And so it gets smaller and smaller until the transmitted power is so high that the BER gets down to zero and the telecommunications are just perfect.
- c) But the surprise is that... for the Sun-Alpha Cen direct radio bridge exploiting both the two gravitational lenses, this minimum transmitted power is incredibly... small ! Actually it just equals less than 10^{-4} watts, i.e. one tenth of a milliwatt is enough to have perfect communication between the Sun and Alpha Cen through two 12-meter FOCAL spacecraft antennas. How is that possible?
- d) Well, that is the “miracle” given to Humanity by the Gravitational Lenses to both explore the universe and keep the link with other stars, you know! Just remember that, in 2009, the discovery of the first extrasolar planet in the Andromeda galaxy (M31) was announced because of the gravitational lens caused by something in between!

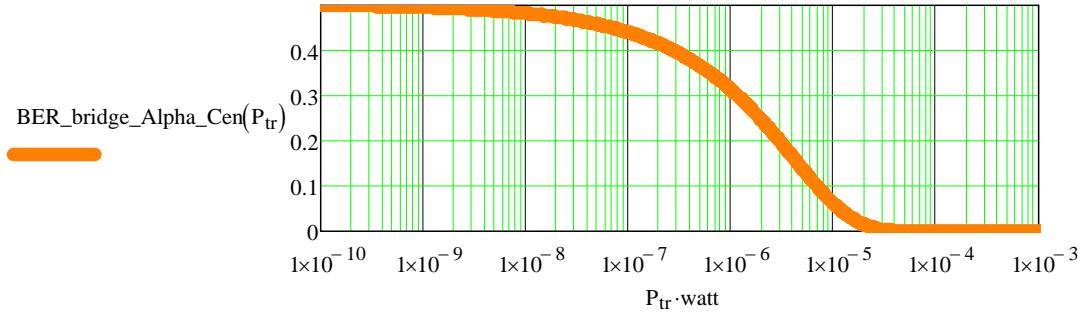


Figure 7. Bit Error Rate (BER) for the double-gravitational-lens system giving the radio bridge between the Sun and Alpha Cen A. In other words, there are two gravitational lenses in the game here: the Sun one and the Alpha Cen A one, and two 12-meter FOCAL spacecrafts are supposed to have been put along the two-star axis on opposite sides at or beyond the minimal focal distances of 550 AU and 749 AU, respectively. This radio bridge has an OVERALL GAIN SO HIGH that a miserable 10^{-4} watt transmitting power is sufficient to let the BER get down to zero, i.e. to have perfect telecommunications! Fantastic!

Notice also that the scale of the horizontal axis is logarithmic, and the trace is yellowish since the light of Alpha Centauri A is yellowish too. This will help us to distinguish this curve from the similar curve for the Barnard's Star, that is a small red star 6 light years away, as we study in the next section.

9. THE “RADIO BRIDGE” BETWEEN THE SUN AND BARNARD’S STAR USING THE TWO GRAVITATIONAL LENSES OF BOTH

The next closest star to the Sun beyond the triple Alpha Centauri system is Barnard's star (see, for instance, http://www.en.wikipedia.org/wiki/Barnard's_Star). Let us now repeat for the gravitational lens of Barnard's star the same calculations that we did in the previous section for Alpha Centauri A. Then one has:

$$\begin{cases} d_{Barnard} = 5.98 \text{ light years} \\ r_{Barnard} = 0.17 r_{Sun} \\ M_{Barnard} = 0.16 M_{Sun} \end{cases} \quad (39)$$

Barnard's star is thus just a small red star, that is actually “passing by” the Sun right now and is not

known to have planets around it, As a consequence of the numbers listed in (39), one infers that

$$\begin{cases} d_{focal_Barnard} = 98 \text{ AU} \\ r_{Schwarschild_Barnard} = 0.47 \text{ km} \\ G_{Barnard}(v_{Ka}) = 1991174. \end{cases} \quad (40)$$

Especially the gain is important to us:

$$\boxed{G_{Barnard}(v_{Ka})=63 \text{ dB}} \quad (41)$$

We replace this into the Barnard's star equivalent of (38), again supposing that two 12-meter FOCAL spacecraft antennas are placed along the Sun-Barnard straight line at or beyond 550 AU and 100 AU, respectively. The result is the new graph of the BER as a function of the transmitted power only as in Figure 8.

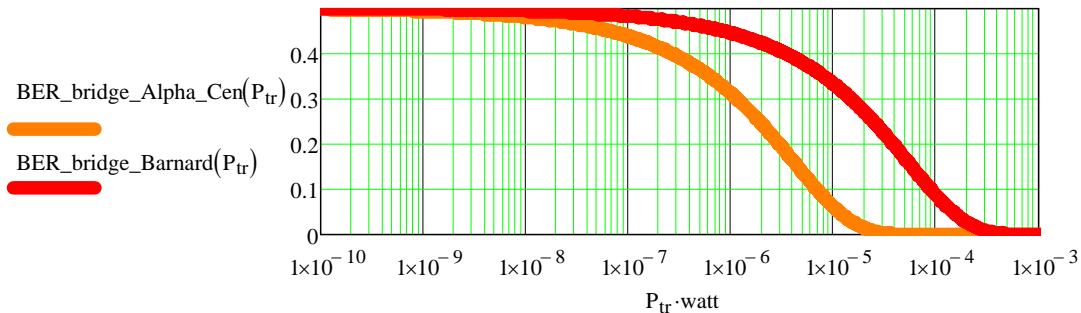


Figure 8. Bit Error Rate (BER) for the double-gravitational-lens of the radio bridge between the Sun and Alpha Centauri A (yellowish curve) plus the same curve for the radio bridge between the Sun and Barnard's star (reddish curve, just as Barnard's star is a reddish star): for it, 10^{-3} watt are needed to keep the BER down to zero, because the gain of Barnard's star is so small when compared to the Alpha Centauri A's.

10. THE “RADIO BRIDGE” BETWEEN THE SUN AND SIRIUS-A USING THE TWO GRAVITATIONAL LENSES OF BOTH

The next star we want to consider is Sirius A. This is because Sirius A is a big, massive bluish star and so it is completely different from both Alpha Cen A (that is a Sun-like star) and Barnard’s star, that is a small red star. Data may again be taken from the Wikipedia site: <http://www.en.wikipedia.org/wiki/Sirius> and one gets:

$$\begin{cases} d_{Sirius_A} = 8.6 \text{ light years} \\ r_{Sirius_A} = 1.711 r_{Sun} \\ M_{Sirius_A} = 2.02 M_{Sun} \end{cases} \quad (42)$$

From these data one gets:

$$\begin{cases} d_{focal_Sirius_A} = 793 \text{ AU} \\ r_{Schwarschold_Sirius_A} = 5.96 \text{ km} \\ G_{Sirius_A}(v_{Ka}) = 251385723 \end{cases} \quad (43)$$

The important thing is of course the gain:

$$\boxed{G_{Sirius_A}(v_{Ka}) = 74 \text{ dB}} \quad (44)$$

Then, one replaces this into the Sirius A equivalent of (38), again supposing that two 12-meter FOCAL spacecraft antennas are placed along the Sun-Sirius A straight line at or beyond 550 AU and 793 AU, respectively. The result is the new graph of the BER as a function of the transmitted power only as in Figure 9.

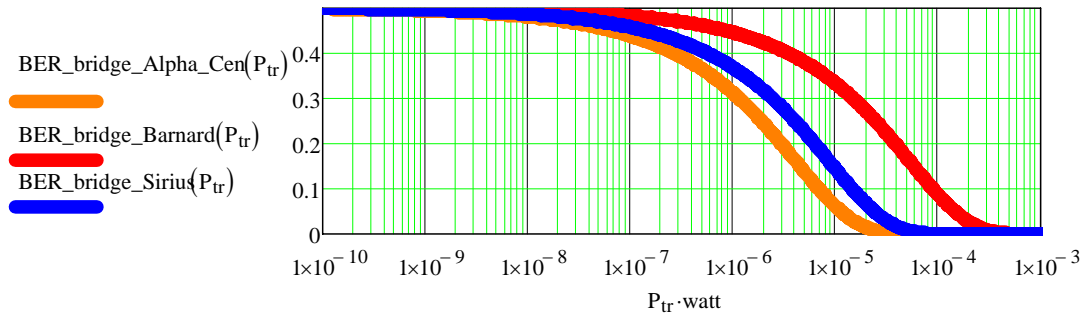


Figure 9. Bit Error Rate (BER) for the double-gravitational-lens of the radio bridge between the Sun and Alpha Cen A (yellowish curve) plus the same curve for the radio bridge between the Sun and Barnard’s star (reddish curve, just as Barnard’s star is a reddish star) plus the same curve of the radio bridge between the Sun and Sirius A (blue curve, just as Sirius A is a big blue star). From this blue curve we see that only 10^{-4} watt are needed to keep the BER down to zero, because the gain of Sirius A is so big when compared the gain of the Barnard’s star that it “jumps closer to Alpha Cen A’s gain” even if Sirius A is so much more further out and the Barnard’s star! In other word, the star’s gain and the size combined matter even more than its distance!

11. THE “RADIO BRIDGE” BETWEEN THE SUN AND ANOTHER SUN-LIKE STAR LOCATED AT THE GALACTIC BULGE USING THE TWO GRAVITATIONAL LENSES OF BOTH

Tempted by the suggestion to increase the distance of the second star more and more, and then see what our calculations yield, we now imagine that the second star is Sun-like (i.e. that it has the same radius and mass exactly as the Sun) but is located... inside the Galactic Bulge! Namely 26,000 light years away, according to the Wikipedia “Milky Way Galaxy” site: http://www.en.wikipedia.org/wiki/Milky_Way . So, the equivalent of (24) and (38) now becomes

$$P_r = \frac{P_t (G_{Sun_at_Ka})^2 (G_{12_meter_antenna_at_Ka})^2}{L(r, \lambda)} \quad \dots(45)$$

and the plot of the BER vs. transmitted power is shown in Figure 10 as the new, pink curve at the far right of the previous three curves of Alpha Cen A (orangish), Barnard’s star (red) and Sirius A (blu). The new pink curve showing the BER of a Sun-like star at the Galactic Bulge is naturally much to the right of the previous three stellar curves inasmuch as the Bulge distance of 26,000 light is so much higher than the distances of the three mentioned nearby stars (less then 10 light years away anyway). The horizontal axis scale is much higher now, since the pink BER curve gets to zero only for transmitted power of about 1000 watt.

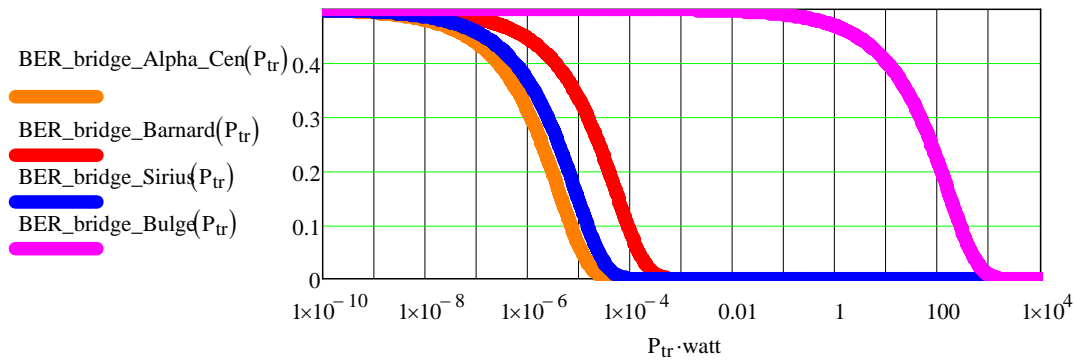


Figure 10. Bit Error Rate (BER) for the double-gravitational-lens of the radio bridge between the Sun and Alpha Cen A (orangish curve) plus the same curve for the radio bridge between the Sun and Barnard’s star (reddish curve, just as Barnard’s star is a reddish star) plus the same curve of the radio bridge between the Sun and Sirius A (blue curve, just as Sirius A is a big blue star). In addition, to the far right we now have the pink curve showing the BER for a radio bridge between the Sun and another Sun (identical in mass and size) located inside the Galactic Bulge at a distance of 26,000 light years. The radio bridge between these two Suns works and their two gravitational lenses works perfectly (i.e. BER = 0) if the transmitted power higher than about 1000 watt.

12. THE “RADIO BRIDGE” BETWEEN THE SUN AND ANOTHER SUN-LIKE STAR LOCATED INSIDE THE ANDROMEDA GALAXY (M 31) USING THE TWO GRAVITATIONAL LENSES OF BOTH

We conclude this paper by calculating the radio bridge between the Sun and another Sun... in Andromeda! The distance is now 2.5 million light year, but the bridge would still work if the transmitted power

was of higher than about 10^7 watts = 10 Megawatt. This is shown by the new cyan curve on the far right in Figure 11 below. Perhaps this idea is not as “crazy” as it might appear, if recently (June 2009) the first extrasolar planet in the Andromeda galaxy was announced to have been discovered just by gravitational lensing. See, for instance, the web site: http://www.redorbit.com/.../possible_planet_found_outside_our_galaxy/index.html

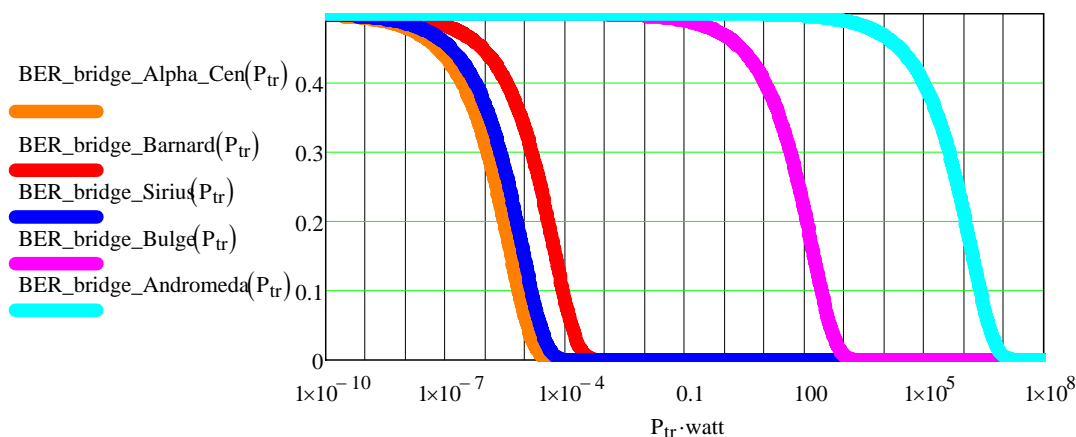


Figure 11. The same four Bit Error Rate (BER) curves as shown in Figure 10 plus the new cyan curve appearing here on the far right: this is the BER curve of the radio bridge between the Sun and another Sun just the same but located somewhere in the Andromeda Galaxy M 31. Notice that this radio bridge would work fine (i.e. with BER = 0) if the transmitting power was at least 10^7 watt = 10 Megawatt. This is not as “crazy” at it might seem if one remembers that recently (June 2009) the discovery of the first extrasolar planet in the Andromeda Galaxy was announced, and the method used for the detection was just GRAVITATIONAL LENSING !

CONCLUSION

In these few pages we could just sketch the FOCAL space mission to 550 AU and beyond to 1000 AU. A number of issues still have to be investigated in: 1) the science related to the mission, 2) in the propulsion tradeoffs to get there in the least possible time and 3) in the optimization of the telecommunication link. Nevertheless it plainly appears that the Sun focus at 550 AU is the next most important target that humankind must reach in order to be prepared for the successive and more difficult task of achieving the interstellar flight. In particular, we proved that the FOCAL mission only will allow us to EXTEND our telecommunications with spacecrafts located in space at least at the distance of Alpha Centauri or higher. This we did by resorting to the notion of Bit Error Rate, that would be zero or nearly zero only if we build up RADIO BRIDGES between the Solar System and the destination target whether that would be a spacecraft or another star system.

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